EXHIBIT G

Understanding the Link between Patent Value and Citations: Creative Destruction or Defensive Disruption?

David S. Abrams
University of Pennsylvania

Ufuk Akcigit
University of Pennsylvania & NBER

Jillian Popadak
University of Pennsylvania

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Abstract

The patent system is the leading legal mechanism for protecting new inventions and as such, patents are used in a host of research to proxy for innovative activity. Understanding how new products and processes are created and how to value them is critical to fields as diverse as industrial organization, endogenous growth theory, and intellectual property law. In this paper we provide the first evidence that much of the work in these literatures is based on an erroneous assumption: that the value of innovation is proportional to citation-weighted patent counts. Using a proprietary dataset with patent-specific revenues, we find that there is an inverted-U relationship between patent value and citations. We attempt to explain this relationship using a simple model of firms, allowing for both productive and defensive patents. Simulations from the model match the empirical regularity that some very high-value patents receive substantially fewer citations than less valuable patents. Further, we find evidence of greater use of defensive patenting along the dimensions where it is predicted. These findings have important implications for our basic understanding of growth, innovation, and intellectual property policy.

JEL Codes: O3, L2, K1.

Keywords: Productive innovation, Defensive innovation, Patents, Creative Destruction, Citations, Patent Value, Competition, Intellectual Property, Entrepreneurship.

1 Introduction

One of the core questions of economics, both at the micro and macro level, is what leads to productivity gains. In order to understand what policies impact innovative activity and ultimately productivity, it is crucial to start with a good metric to value innovation. While the importance of such a metric has long been recognized (Scherer 1956; Grilliches 1990) so too have the inadequacies of the proxies for value that are in widespread use (Schankerman and Pakes 1986; Hall and Harhoff 2012).

Over the last 30 years, two primary metrics have been used to proxy for the value of innovation, patent counts and citation-weighted patent counts. The intuition is simple: fields with greater innovative activity will have more value to protect and will do so by applying for more patents. Weighting patent counts by forward citations[1] is a natural augmentation to simple patent counts, given the well-known fact that patents vary tremendously in value[2]. This metric, however, is based on the assumption that a larger number of citations corresponds to higher value.

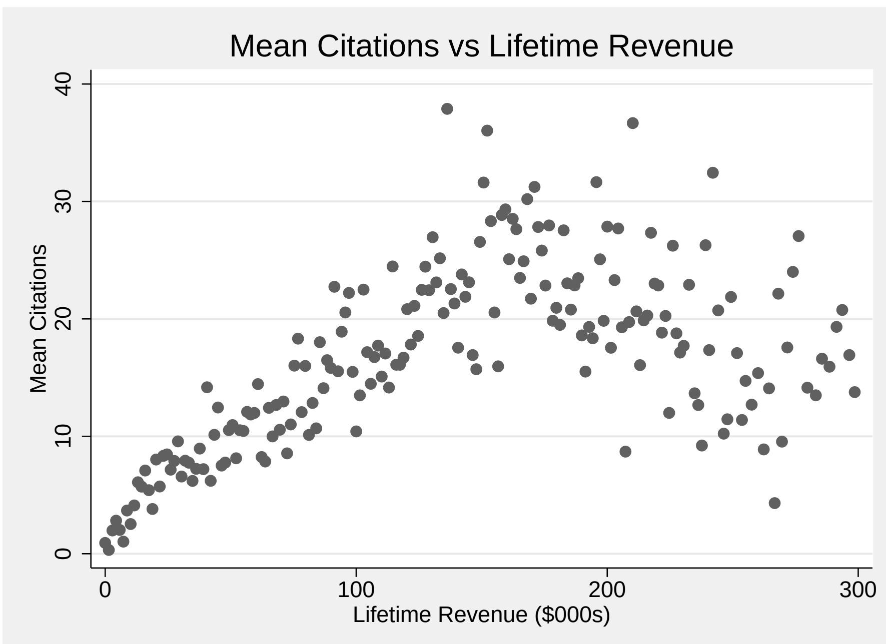

Figure 1: LIFETIME FORWARD CITATIONS VS. REVENUE
Notes: Data is normalized so that the mean annual revenue is $10,000.

Yet, the history of science and economics is replete with theories that did not bear up

[1]Forward citations is the number of citations received by a particular patent by subsequent patents.

[2]Fewer than 10 percent of patents are worth the money spent to secure them (Allison, Lemley, Moore, Trunkey 2009), but the most valuable ones are thought to be worth hundreds of millions of dollars (Hall, Jaffe, and Trajtenberg 2005).

under empirical scrutiny and until now there has been no good way to test this assumption. In order to say anything convincing about innovation we need a credible measure of its value. In Figure 1 we present strong evidence that the main approach to valuing innovation is fatally flawed. The relationship between citations and patents is not only non-linear, it is not even monotonic. This striking finding calls for a deeper understanding of the process of innovation, patenting, and citations, which we explore empirically and theoretically in this paper.

The citation-value relationship revealed in Figure 1 is extremely surprising relative to what has previously been assumed. Prior empirical study of the relationship was quite limited due to several problems: companies are reluctant to share proprietary patent data, single firm portfolios tend to have limited technological breadth and small sample size, and almost no companies allocate revenues to specific patents. This paper is only possible by virtue of access to a very large, diversified patent portfolio owned by non-practicing entities (NPEs) that calculate patent-specific revenues. We discuss details of the data set and its advantages for academic inquiry further in Section II.

We introduce a theoretical model that suggests that the inverted-U shape is the result of two types of innovative effort, which we characterize as productive and defensive. Productive innovative effort leads to the traditional increasing relationship between patent value and citations; defensive innovative effort, however, leads to a negative relationship between patent value and citations. In an economy that exhibits both of these types of innovative effort, the link between patent value and citations will be the inverted-U that we observe empirically.

We test several predictions of the model, besides the overall inverted-U shape. Defensive patenting should be more prevalent among larger entities, for divisional and continuation patents, for newer patents, and in technology classes with rapid growth. Each of these predictions is borne out in the data and we find evidence that defensive patenting is more prominent in these categories.

This is certainly not the first paper that has attempted to examine the relationship between patent value and citations, but it is the first not severely constrained, for the reasons mentioned above. Trajtenberg (1990) is perhaps the leading prior work on the subject, but he had access to a data set several orders of magnitude smaller than in this paper. In addition, all patents were in a single narrow field (computed tomography or CT) and values were imputed from a structural model of the CT device. Harhoff, Scherer, and Vopel (2003), obtain categorical measures of value on 772 patents from a survey of German patents with 1977 priority that were renewed to full term. Several excellent studies examine the patent value distribution using the renewal decision to infer value (Pakes 1986; Schankerman and Pakes 1986; Bessen 2008). These papers make use of the contingent claim valuation method pioneered by Pakes and Schankerman. Since a renewal decision can only convey an upper or lower bound on value, this approach is not useful for learning more about the citation-value relationship.

In the legal literature, defensive patenting has received a great deal of attention in re-

cent years as allowable subject matter has widened to include software and business methods patents. As the number of patents granted has increased, technological progress has led to devices that implicate thousands of separate patents. Some have argued that we have arrived at a point where the patent system is actually detrimental to innovation (Bessen and Meurer 2008; Boldrin and Levine 2012). We capture these observations and intuitions by modeling defensive patents as ones which do not lead to substantial further work in a field and in fact may stifle it (blocking patents). Thus, there may be extremely valuable defensive patents that receive very few citations, leading to a null or negative relationship between forward citations and revenue.

A single figure is not enough to convince one of the correctness of a theory, or even of the robustness of the empirical findings. We aim to tackle both of these tasks in the balance of the paper, but we take the unusual step of including this striking figure in the beginning because it immediately conveys our central contribution. In Section II we provide substantial detail about incentives to patent and cite, the business models of NPEs and further description of the data. Section III introduces our model which we believe captures some of the key elements of the patenting and citing processes. In Section IV we present the main empirical results and a discussion of them. Section V concludes and makes the point that the goal of this work is not to undermine the large body of work on innovation that has relied on widely-held assumptions about the patent value-citations relationship. Rather, we hope that this will help build a more robust literature that informs some of the central economic issues of our time.

2 Background

Since the major limitation of previous studies of patent value is due to the lack of available data on individual patent revenues, it is worth discussing the data source and characteristics in some detail. The data in this paper was provided by large non-practicing entities (NPEs), with focuses in the technology sectors. NPEs are firms whose revenue primarily derives not from producing products based on patented technology, but from licensing patents. These companies employ a range of different business models ranging from aggressive litigators to passive licensors, and the number of patents held by NPEs continues to grow rapidly.

This is fortunate for those interested in learning about innovation as NPEs function as an excellent data source in many ways, and when compared to traditional patent holding firms, NPEs have several advantages as an object of study. Their portfolios can be substantially larger than practicing firms, since their capital is almost exclusively employed in assembly and licensing, rather than production. NPEs are more diversified than practicing firms as well, since it is often easier to acquire the breadth of expertise necessary to acquire and license patents in a large array of fields, rather than to practice them. The data available from NPEs is also likely to be substantially more useful for researchers, as they tend to determine patent-

specific revenues. This is something that almost no practicing firms do, unless licensing is a major part of their business. This should not be surprising since ultimately most firms care about overall profit from innovation, not specifically from which patent the profit derives.

Table I reports variables definitions and summary statistics for the primary patent and assignee characteristics analyzed in this paper. After dropping design and plant patents, we observe 46,891 regular, utility patents. The average lifetime patent value is $204,212, but the standard deviation is $1.9 million. The mean number of forward citations is 13, but the median is 0. This degree of skewness in the distributions of patent value and forward citations is similar to that reported by Trajtenberg (1990); Harhoff, Scherer, and Vopel (2003); and Bessen (2008).

The heterogeneity in the underlying patent characteristics and assignees is extensive. The patents are licensed to and acquired from a broad range of intellectual property sources including individual inventors, small firms, large firms, universities, hospitals, and government agencies. The dataset represent patents originated in 89 different countries, and patents granted in the United States represent just less than the majority at 46 percent. Individual inventors account for 58% of the patents, and the average patent has 2 inventors that make 20 claims, of which 16 are dependent claims. On average, backward citations are not concentrated in very recent patents with only 20% in the three years prior to application.

Table II describes the diverse range of technologies that are patented. Our sample covers 267 unique primary technology classifications, which we have grouped into 10 broad technology categories. The technology categories include: internet and software, wireless communications, circuits, network communications, computer architecture, peripheral devices, semiconductors, electromechanical, optical networking, and nanotechnology.

In our subsequent theoretical and empirical analyses, where we attempt to provide a theoretical foundation for the inverted U-shape in the data, we focus on a few variables characterized by productive and defensive innovations. While building our theoretical model, we rely on the *Schumpeterian theory of creative destruction* (see the recent survey by Aghion, Akcigit and Howitt (2013) for more on this topic), where each new innovation builds on previous technologies, but also makes them obsolete by introducing a better one. This tension between the incumbent technology owner's wish to defend its monopoly power and the future innovator's wish to utilize the spillovers generated by the current incumbent help us rationalize the non-monotonic relationship between patent value and subsequent entry, identified by forward citations. Moreover, models presented by Farrell and Shapiro (2008) emphasize the ability of patent holders, even of weak or less productive patents, to hold up firms through the threat of infringement. Similarly, our model emphasizes the decision to innovate productively or defensively. Intuitively, this suggests that non-original and less productive patent applications with a higher concentration of backward citations in recent years are more likely to be strategic or

defensive patents. Around 16% of the patents in our sample are non-original[3] and only 20% of the backward citations are in the recent past.

Since a major contribution of this paper is a better understanding of the relationship between patent value and citations, it is important to clearly define how those are calculated in this paper. The NPEs from which the data are derived purchase or entering into revenue sharing agreements with patent owners. Revenue is generated by licensing the patents in the entire NPE's portfolio or a subset of a NPE's portfolio. Revenue is allocated on a patent-year-customer level based on the prominence the patent played in negotiations with the customer. This allocation scheme is disciplined by competing interests on two sides. Patent owners who are due a share of future revenues seek to maximize the revenue allocated, while the incentive of shareholders in the NPE is for larger revenue allocation to patents in which they have a stake and less to others, since total revenue allocation is a zero-sum game. We aggregate revenues to the patent-year level and then compute the mean revenue profile over the life of a patent separately for each of the 10 primary technology categories. We estimate lifetime revenue for each patent by inflating the observed revenue by the ratio of lifetime revenue to the mean of the years we observe for each patent. We then normalize all revenue amounts so that mean annual revenue is $10,000 in order to maintain the confidentiality of the revenue data.

Lifetime citations are computed in a similar manner. We obtain data on forward citations, defined as the total number of times a patent has subsequently been cited. By definition, newer patents will have less time to acquire citations than old ones and this must be accounted for. We define "lifetime citations" as the total number of citations we expect a patent to have by its expiration. We compute this by first producing the forward citation- patent age profile for each of our ten technology categories. Figure 2 presents the incremental patent citation profile and an associated revenue profile on aggregate. There is substantial variation by technology class; therefore, we create separate revenue and citation profiles for each technology class. We calculate lifetime citations by inflating the total citations already received by the ratio of the total mean citations divided by the mean for the average patent of the same age as the one in question. One small flaw in this procedure is that it will understate the number of lifetime citations for any patent that has zero in our dataset, but the mean number of lifetime cites should still be correct.

[3] Within the intellectual property legal framework, an original patent is an application that establishes its own filing date and does not have an effective filing date based upon another previously filed application. If an "original" application is then used to establish an effective filing date of a later filed application, it becomes known as a parent application and the later filings are either divisions or continuations. There can be many strategic advantages to non-original patents if the first-to-file is important or if one desires to prolong the original patents disclosure.

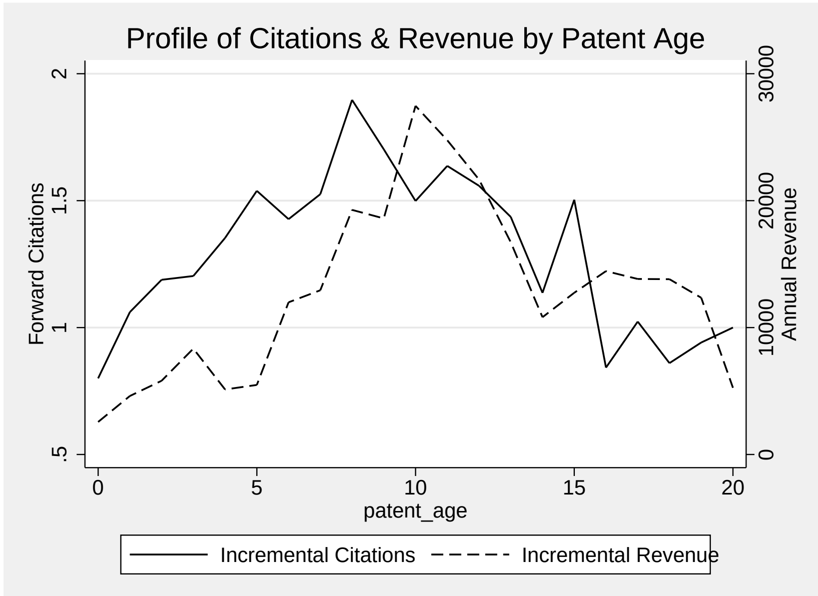

Figure 2: INCREMENTAL FORWARD CITATIONS AND REVENUE BY PATENT AGE
Notes: Data is normalized so that the mean annual revenue is $10,000.

3 Theory of Patent Valuations and Citations

In the previous section, we provided a striking new empirical finding which is at odds with the received wisdom about the link between patent value and citations. How can we reconcile the two and account for the inverted-U? In this section, we offer a new model of innovation, patents, and citations. Our purpose is to develop a better understanding of the underlying reasons for the observed inverted-U relationship between citations and patent value. We embed intuitive assumptions into a structural model, and show that the model fits the observed pattern well.

Our model features two distinct types of innovation efforts – *productive* and *defensive*. The intuition for productive innovation follows the traditional economic view that patents are offered as a contract between society and the inventor. In return for a limited period of exclusivity, the inventor agrees to make his invention public rather than keeping it secret. This institutional arrangement promotes the diffusion of ideas and economic growth. However, this is likely not the full story. Therefore, we also introduce the notion of the defensive innovation, a type of destructive creation. This idea seeks to capture the fact that when firms and individuals are endowed with a complex legal instrument, they may use it strategically in ways that do not serve the original intent of the legislation that created the instrument in the first place.

To help put some structure on these two types of innovative effort, we develop a model. For reasons that we explain below, our model predicts that the link between patent value and citations are positive for productive innovation efforts and negative for destructive innovation

efforts. The combination of the two cases generates the inverted-U relationship that is so prominent in the data. One of the reasons for approaching this problem from a structural paradigm is that it will allow us to quantify a number of crucial moments such as the size of the creative production and non-creative destruction. Further, given the properties of the decentralized market that we embed in the model, we will be able to make welfare statements such as what the impact of a counterfactual innovation policy may be. Thus, this type of exercise leads to practical findings for researchers, practitioners, and policymakers alike.

3.1 The Case of Productive Innovations

In this section, we introduce a continuous-time model with a representative household. The household consumes a basket of goods, each of which is produced by a different incumbent monopolist. The economy features a large number of outside entrepreneurs who invest in productive innovations. These productive innovations enable the entrepreneurs to innovate, to replace existing incumbents, and to obtain market share. In the first model with productive innovations, we abstract from incumbent innovations and focus only on entrants' innovations. This assumption is relaxed in the subsequent model where we allow incumbent firms to create defensive innovations, which protect their valuable productive patents and market share.

The key feature of the productive innovation model that relates to citations is how new innovations arrive. Specifically, we assume that new innovations and innovative efforts arrive in clusters and that each new patent cites the prior art within the same technology cluster. Intuitively, certain markets become hot and attract all the top talent to invest their innovative efforts in that market. This simple logic leads to clustering of innovations by technology sector over time . Although this is an assumption, it is also consistent with empirical evidence (Jaffe and Lerner 2004). In terms of the model, what follows from this logic is an endogenous-citation dynamic.

The link between the citations and patent value comes from the fact that more novel innovations will have larger mark-ups due to their originality, denoted by the step size of a new innovation. In the model, this then translates into larger patent values. Thus, the first simple model of productive innovation effort leads to the traditional conclusion of a positive correlation between patent citations and patent value. At the same time, more novel innovations will generate larger spillovers for the subsequent innovations, which will encourage new innovations by outside entrepreneurs. With more entrepreneurs entering the market, a natural cluster of innovative effort over time by technology is created. Since a new innovation must cite the previous related patents upon which it builds, more novel patents receive more citations on average. Given the intuition and logic underlying this first model of productive innovation, we now turn to the details.

Basic Environment Consider the following continuous time economy that admits a representative household. The household consumes a unique consumption basket C_t that consists of large set of varieties indexed by $j \in [0, 1]$ as follows:

$$C_t = \exp \int_0^1 \ln c_{jt} dj, \tag{1}$$

In this expresssion, c_{jt} is the quantity of variety j at time t. We normalize the price of the final good C_t to be 1 in every period without loss of generality. The consumption basket is produced in a perfectly competitive market.

Each variety j is produced by a monopolist who owns the latest innovation (patent) in sector j. The monopolist's production function takes the following simple form

$$c_{jt} = q_{jt} l_{jt} \tag{2}$$

where l_{jt} is the labor employed for production and q_{jt} is the variety-specific labor productivity. In what follows, new innovations will improve labor productivity, which leads to an aggregate growth in this economy. The linear production function implies that the marginal cost of producing 1 unit of c_{jt} is simply

$$M_{jt} = \frac{w_t}{q_{jt}}$$

where w_t is the market wage rate which is taken as given by the firm. Note that all monopolists hire from the same labor market in the economy, hence every monopolist faces the same wage rate w_t.

Labor productivity q_{jt} is improved through subsequent innovations in each product line j. Innovations belong to technology clusters. Let n index the order of an innovation in a technology cluster such that the very first patent that starts a new technology class has $n = 0$, the first follow-on innovation in the same technology cluster is indexed by $n = 1$, the second follow-on innovation by $n = 2$, and so on. Each innovation by a new entrant into j improves the previous incumbent's technology by a factor of $(1 + \eta_n)$ which is only a function of the order n of the patent in the technology class and remains constant as long as the same firm is in charge of production. Consider a product line where productivity at time t is q_{jt} and a new innovation of step size η_n is received during $(t, t + \Delta t)$. Then the labor productivity evolves as:

$$q_{jt+\Delta t} = (1 + \eta_n) q_{jt}. \tag{3}$$

When a new firm innovates and enters into j as the new market leader, the latest innovator and the previous incumbent compete in prices à la Bertrand.

3.1.1 Static Equilibrium: Production, Pricing and Profits

It is useful to solve the static production and pricing decisions before we describe the innovation technology. Consider the consumption basket in (1). Because the consumption basket has a Cobb-Douglas form with respect to all varieties, the household will spend the same amount C_t on each variety j. Hence the demand for each variety j can be expressed as

$$c_{jt} = \frac{C_t}{p_{jt}} \tag{4}$$

where p_{jt} is the price charged by the monopolist j. Note that the Bertrand competition between the new monopolist and the previous incumbent, together with the unit elastic demand curve in (4) implies that the monopolist will follow limit pricing and charge a price that is equal to the marginal cost of the previous incumbent. If the productivity of the current monopolist in j is q_{jt} and the size of her innovation was η_n, then the marginal cost of the previous incumbent is simply $(1+\eta_n)\, w_t/q_{jt}$, which implies that the current monopolist's price is simply

$$p_{jt} = \frac{(1+\eta_n)\, w_t}{q_{jt}}.$$

Therefore we can express the equilibrium profit of the monopolist j as

$$\begin{aligned} \pi_t(q_{jt}) &= [p_{jt} - M_{jt}]\, c_{jt} \\ &= \pi_n C_t \end{aligned}$$

where we define $\pi_n \equiv \frac{\eta_n}{1+\eta_n}$ as the normalized profit $(= \pi_t(q_{jt})/C_t)$. This is the first step in establising the value of an innovation. Because a new innovation grants a patent protection until another new innovation makes it obsolete through creative destruction, the value of an innovation (patent) will be the expected sum of future monopoly profits that will be generated by this innovation.

The following lemma summarizes the rest of the static equilibrium variables C_t and w_t.

Lemma 1 *The aggregate consumption in this economy is equal to*

$$C_t = Q_t$$

where Q_t is defined as a productivity index

$$Q_t \equiv \left[\int_0^1 (1+\eta_j)^{-1}\, dj\right]^{-1} \exp \int_0^1 \ln \frac{q_{jt}}{1+\eta_j} dj.$$

Moreover, the wage rate is equal to

$$w_t = Q_t \int_0^1 (1 + \eta_j)^{-1} \, dj.$$

3.1.2 R&D and Productive Innovations

The economy has a measure of outside entrepreneurs who try to innovate and replace the existing incumbents. Outside entrepreneurs invest in R&D to produce a new innovation stochastically. When they are successful, they improve the latest quality as in (3) . However productive innovations come in clusters as in Akcigit and Kerr (2010). In particular, new entrants invest in two types of innovations:

1. *radical innovations,*
2. *follow-on innovations.*

When a new radical innovation occurs, it re-starts a new technology cluster with a step size $\eta_0 = \eta > 0$. Alternatively, if a new follow-on innovation occurs, it directly builds on the existing technology and the marginal contribution of this new innovation depends on how exploited the technologies are within the same technology cluster. In other words, follow-on innovations run into dimishing returns within the cluster such that the n^{th} follow-up innovation has a step size of $\eta_n = \eta\alpha^n$ where $\alpha \in (0, 1)$. For mathematical convenience, we assume that after a certain number of follow-on innovations ($n > n^*$), the step size becomes a constant value $\eta_n = \eta\alpha^{n^*}$. In summary, the step size of the $n+1^{st}$ patent in a given technology cluster can be summarized as follows:[4]

$$\eta_n = \begin{cases} \eta \text{ if radical innovation} \\ \eta\alpha^n \text{ if follow-on innovation and } n < n^* \\ \eta\alpha^{n^*} \text{ if follow-on innovation and } n \geq n^* \end{cases}.$$

Since innovations come in technology clusters and that each new innovation utilizes the spillover from the previous patents from the same technology class, our model generates a natural interpretation of citations. When there is a major innovation in a technology class with a step size η, it produces spillovers for the subsequent innovations since the follow-on step size becomes $\eta\alpha$ which encourages new entry into the field. Innovations must cite previous innovations within the same technology cluster, acknowledging that the patents are technologically related. Therefore, patents from the same technology cluster will cite the initial major patent that opened the field. The following example will elaborate this structure further.

[4]Note that in principle, we can allow the step size η_j to be a function of the sector j. This would not have any major impact on the inverted-U relationship that our model predicts.

Example 1 *This example is provided to show the connection between our model and the data. In particular, we describe how technology clusters emerge and who cites who in those clusters. The following chart illustrates an example of some innovation patterns in a single product line:*

$$\left|\ \underbrace{\frac{\eta}{P_1}\,\frac{\eta\alpha}{P_2}\,\frac{\eta\alpha^2}{P_3}\,\frac{\eta\alpha^3}{P_4}}_{\textit{Tech Cluster 1}}\ \right|\ \underbrace{\frac{\eta}{P_5}\,\frac{\eta\alpha}{P_6}}_{\textit{Tech Cluster 2}}\ \left|\ \underbrace{\frac{\eta}{P_7}\,\frac{\eta\alpha}{P_8}\,\frac{\eta\alpha^2}{P_9}}_{\textit{Tech Cluster 3}}\ \right|\ \underbrace{\frac{\eta}{P_{10}}}_{\textit{Tech Cluster 4}}\cdots$$

An example of a sequence of innovations in a product line

Example starts with a radical innovation P_1 *which has a step size* η. *Then innovation* P_2 *follows on* P_1 *with a step size* $\eta\alpha$. *Since* P_3 *is the second follow-on innovation in cluster 1, it has a step size* $\eta\alpha^2$ *and so on. Note that* P_5, P_7 *and* P_{10} *turn out to be a radical innovations which start new technology clusters; therefore their step sizes are* η. *As a result, innovation step sizes follow cycles. Finally, the citing-cited pairs can be summarized as follows:*

Cited	***Citing***	***Cited***	***Citing***
$P_1:$	P_2, P_3, P_4	$P_6:$	*none*
$P_2:$	P_3, P_4	$P_7:$	P_8, P_9
$P_3:$	P_4	$P_8:$	P_9
$P_4:$	*none*	$P_9:$	*none*
$P_5:$	P_6	$P_{10}:$	...

Consider P_2, *for instance. Since it builds only on* P_1, P_2 *cites only* P_1. *However, there are two patents* (P_3, P_4) *in the cluster that are building on* P_2. *Hence,* P_2 *receives two citations from them.*

Now we can turn to the value of an innovation. Consider an innovation of step size $\eta_n = \eta\alpha^n$. Let the aggregate innovation arrival rate of the next follow-on innovation be denoted by $\bar{z}_{n+1}$ and the next radical innovation by $\bar{z}_0$. Then the steady-state value of the n^{th} innovation is summarized by the following continuous time Hamilton-Jacobi-Bellman (HJB) equation

$$V_{nt} = \frac{\eta_n}{1+\eta_n} C_t \Delta t + (1 - r\Delta t) \left[\begin{array}{c} (\bar{z}_0 \Delta t + \bar{z}_{n+1} \Delta t) \times 0 \\ + (1 - \bar{z}_0 \Delta t - \bar{z}_{n+1} \Delta t) V_{nt+\Delta t} \end{array} \right].$$

This expression is intuitive. During a small Δt, n^{th} innovation in a cluster delivers a profit of $\frac{\eta_n}{1+\eta_n} C_t \Delta t$ to its owner. The future period is discounted by $(1 - r\Delta t)$. After Δt, with probability $\bar{z}_{n+1}\Delta t$ there is a new follow-on entry, and with probability $\bar{z}_0 \Delta t$ there is a radical entry. In both cases, the incumbent exits the market becuase she is replaced by a new entrant and her firm value decreases to 0. With the remaining probability $(1 - \bar{z}_{n+1}\Delta t - \bar{z}_0 \Delta t)$, the

incumbent survives the threat of entry and receives the continuation value $V_{t+\Delta t}$ of being the incumbent. Subtracting $(V_{nt+\Delta t} - r\Delta t V_{nt})$ from both sides, dividing through Δt, and taking the limit $\Delta t \to 0$ leads to the following HJB equation:

$$rV_n - \dot{V}_n = \pi_n C_t - (\bar{z}_{n+1} + \bar{z}_0) V_n. \tag{5}$$

where $\pi_n \equiv \frac{\eta_n}{1+\eta_n}$. The following lemma provides the exact form of the value function.

Lemma 2 *The normalized value of the n^{th} follow-on innovation at time t is equal to*

$$v_n \equiv \frac{V_{nt}}{C_t} = \frac{\pi_n}{\rho + \bar{z}_{n+1} + \bar{z}_0} \tag{6}$$

where $\pi_n \equiv \frac{\eta_n}{1+\eta_n}$.

Proof. This result follows from using the household's Euler equation $r - g = \rho$ in (5) ■

This expression simply says that the value of an innovation depends mainly on four factors: First, a larger step size η_n implies larger mark-up and therefore higher innovation value. Second, if the aggregate consumption C_t is larger, each variety will receive a larger demand and hence generate higher per-period profit and innovation value. Third, present discounted value of future profits depends on growth rate adjusted interest rate $r - g$, which boils down to the discount rate ρ through the household problem. Finally, the rate of creative destruction of the next follow-on innovation $\bar{z}_{n+1}$ or radical innovation $\bar{z}_0$ lowers the value of the current innovation due to shorter expected duration of monopoly power.

So far, we determined the value of each innovation v_n, as a function of the next innovation's arrival rate $(\bar{z}_{n+1} + \bar{z}_0)$. In order to pin down the arrival rate of follow-on innovations and radical innovations, we now turn to the entry problem of outside entrepreneurs. Let z_n denote innovation rate of an individual entrepreneur and $\bar{z}_n$ denote the aggregate innovation rate by the outside entrepreneurs who are trying to innovate in the same product line j. We assume that there are some congestion externalities such that the individual cost of innovation $K(z_n)$ is increasing in the aggregate innovation rate such that

$$K(z_n) = z_n \zeta Q_t \bar{z}_n \text{ for } n \geq 0$$

in terms of the final good and $\zeta > 0$ is some constant. Then the free-entry for a new entrant can be summarized as

$$\max_{z_n} \{z_n v_n C_t - z_n \zeta Q_t \bar{z}_n\}.$$

Free-entry condition pins down the aggregate entry rate as

$$\bar{z}_n = \frac{v_n}{\zeta}. \tag{7}$$

As expected, entry rate is increasing in the value of a new innovation and decreasing in the cost parameter ζ.

Now combining this last expression (7) with (6) gives us the recursive solution of patent value

$$v_n = \frac{\pi_n}{\rho + (v_0 + v_{n+1})/\zeta}.$$

Finally, the limit value of patents with $n > n^*$ is

$$\bar{v} = \frac{\zeta}{2}\left[\sqrt{\left(\rho + \frac{v_0}{\zeta}\right)^2 + \frac{4}{\zeta}\pi_{n^*}} - \left(\rho + \frac{v_0}{\zeta}\right)\right].$$

Here are the main results emerging from this first model:

Proposition 1 *The average number of forward citations received by an η_n patent during any time interval $[t_1, t_2]$ decreases in n.*

Corollary 1 *Hence, in the case of productive patent, patent value and forward citations are* ***positively correlated****.*

The intuition behind this result is very straightforward: when a new path-breaking innovation occurs, it creates a new technology cluster which then generates spillovers for the subsequent innovations. These spillovers generate a large number of entrants which all then cite the prior art in the cluster. Since the path-breaking major innovation also has the largest mark-up (and value, accordingly), the positive correlation follows.

Figure 3 illustrates this positive correlation. We simulate the above model for 50,000 patents for 100 years. On the x-axis, we list the patent valuations and on the y-axis, we have the number of citations that was received by each patent.

3.2 The Case of Defensive Innovations

In the previous model, incumbents were passive in terms of protecting their monopoly position. In this section, we relax this assumption and introduce the possibility of doing *defensive innovations* by the incumbents to secure their position. The idea is that if an incumbent has a high value productive innovation, then she can potentially invest in some defensive innovation in order to make it harder for the next outside entrepreneur to leapfrog and steal the high monopoly rents. If a defensive patent is very successul from the patenting firm's point of view, the probability of being invented on would be very small which would increase the value of a defensive innovation and decrease the expected number of citations received due to lack of entry. Hence, we should expect a negative relationship between patent valuations and citations in the case of defensive patents.

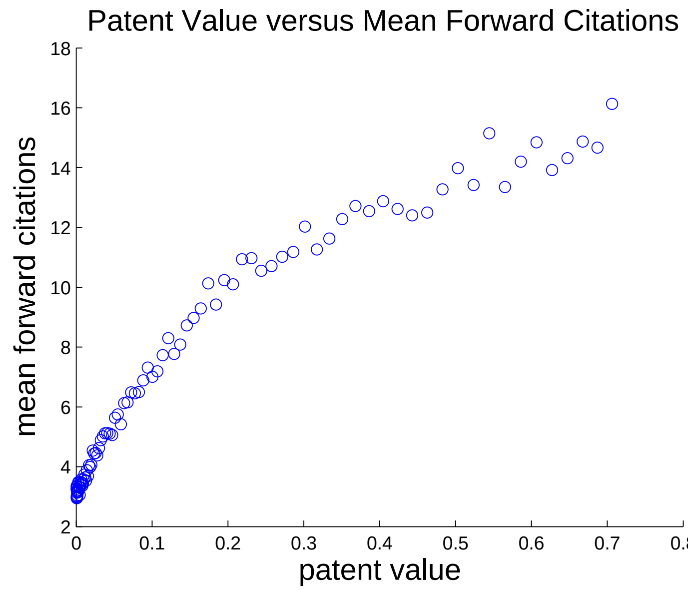

Figure 3: MEAN LIFETIME FORWARD CITATIONS VS. PRODUCTIVE PATENT VALUE

Formally, upon each productive innovations, an incumbent has also the opportunity to do a single defensive innovation. The technology for defensive innovation is such that by paying a fixed cost $\psi > 0$, and new entrant who just invented a productive patent can also obtain a defensive patent. To simplify the analysis, assume that ψ is high enough such that it is profitable to invest in defensive innovations only for the radical inventors (i.e., inventors with step size η). When a firm does defensive innovation, it raises the cost of innovation for the subsequent innovator by a multiplier $m > 1$ which is an iid random variable (realized upon innovation) such that the cost of the next outsider is

$$K(z_{nm}) = \begin{cases} mz_{0m}\zeta Q_t \bar{z}_{0m} \text{ for radical inventors} \\ mz_{1m}\zeta Q_t \bar{z}_{1m} \text{ for follow-on inventors} \end{cases}.$$

Consider the value v_m^d of an $m-$type defensive patent. Since a defensive patent is done only by radical inventors, the profit collected every instant is π_0. Therefore the HJB equation is simply $\rho v_m^d = \pi_0 - (\bar{z}_{0m} + \bar{z}_{1m}) v_m^d$. This value function is expressed as

$$v_m^d = \frac{\pi_0}{\rho + \bar{z}_{0m} + \bar{z}_{1m}}. \tag{8}$$

Now consider the free-entry condition of an outsider who tries to enter after a defensive patent

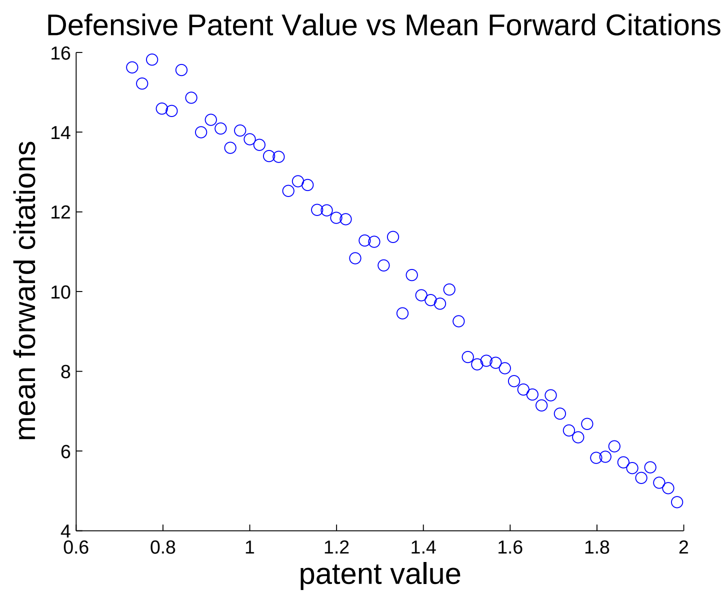

Figure 4: MEAN LIFETIME FORWARD CITATIONS VS. DEFENSIVE PATENT VALUE

of size m. Then for $n \in \{0, 1\}$ the entry problem is simply

$$\max_{z_\kappa} \{z_{nm}v_n - mz_{nm}\zeta\bar{z}_{nm}\}$$

which implies

$$\bar{z}_{nm} = \frac{v_{nm}}{\zeta m}.$$

An important result here is that as the cost of innovation increases through a higher value of m, the entry rate (and the potential forward citation rate) decreases.

Next, combining this entry rate with (8) we get the value of a defensive patent of type m:

$$v_m^d = \frac{\pi_0}{\rho + \frac{v_{0m}+v_{1m}}{\zeta m}}.$$

Now we have the new results.

Proposition 2 *The value of defensive patents increases in m.*

Proposition 3 *The entry rate (forward citations) decreases in m.*

Corollary 2 *Hence, in the case of defensive patents, patent value and forward citations are **negatively correlated**.*

Clearly, the underlying reason for this negative relationship stems from the fact that more successful defensive patents are the ones that increase the cost of entry the most (high m).

When this is the case, the subsequent number of forward citations will decrease due to lower entry. In the meantime, lower entry rate means that the current incumbent can enjoy the monopoly power longer, which raises the value of the defensive patent. Hence we get the negative relationship between defensive patent valuation and citations, as illustrated in Figure 4.

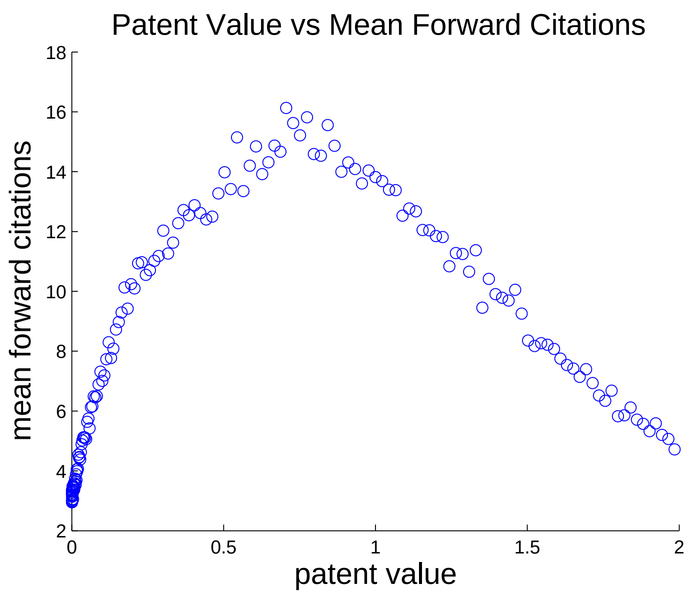

Figure 5: MEAN LIFETIME FORWARD CITATIONS VS. PATENT VALUE

Figure 5 illustrates the overall relationship between patent value and citations. The pattern is a very clear inverted-U, and repeats what we observe empirically in the data. The basic intuition is that the most valuable innovations, which are the radical ones that leapfrog the competition, are owned by entrepreneurs, who are willing to also invest a fixed cost to defensively innovate in order to protect their productive innovation. The combination of the radical, productive innovation and the defensive innovation is very valuable, but because the defensive innovative alters the entry rate of new entrepreneurs through our endogenous citation dynamic, forward citations are dramatically reduced. Put another way, since forward citations enumerate all previous innovations since the most recent radical innovation, the reduction in citations is not due to a less valuable technology, but rather it is due to a more costly entry rate for new entrepreneurs.

3.3 Closing the model: Household Problem

In this section, we close our model by solving the household's maximization problem. The representative household consists of a fixed measure of 1 production workers each of which

supplies one unit of labor inelastically. The household holds a balanced portfolio of assets of all the firms in the economy $\mathcal{A}_t$, earns $r_t\mathcal{A}_t$ from it, collects the labor income w_t and chooses consumption C_t to maximize the following lifetime utility

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt$$

subject to the following budget constraint

$$w_t + r_t\mathcal{A}_t = C_t + \dot{\mathcal{A}}_t.$$

Note that the household discounts the future at the rate $\rho > 0$. Household's intertemporal maximization delivers the standard Euler equation

$$g_t = r_t - \rho. \tag{9}$$

4 Empirical Results

In the previous section we have seen how productive and defensive patents can combine to produce an inverted-U relationship between citations and patent value. We now expand on the empirical results first presented in the introduction and seek to test the model further by examining the relationship for different subsets of the data.

Figure 1 showed an increasing relationship between revenue and citations for values under \$150,000 and then a decreasing relationship for values above this threshold.[5] Table III reports results from the regression analog of the figure: a regression of citations on a quadratic in revenue, with no controls. The three columns in the table vary the share of the overall dataset that is included, winsorizing the top 10, 5, and 1% in columns 1, 2, and 3, respectively. The regression coefficients reinforce the impression from Figure 1, that a concave parabola is a good fit for the data.

No covariates were included in Figure 1, and one may be concerned that variation of both value and citations by these covariates drives the observed relationship. Thus Figure 6 plots forward citations against patent value where citations are demeaned by technology category, individual/corporate inventor, and whether the patent was original. The same general pattern as discussed above is still present, with an increase in citations with patent value up to a point, and then a decrease, although here the relationship is a bit noisier.

Table IV adds the same covariates, along with a dummy for patent age, as controls in the quadratic regression of citations on patent value. The coefficients on the linear and quadratic

[5]The figure excludes the 5% of highest value patents since the long right tail of the value distribution obscures important relationships in the bulk of the data.

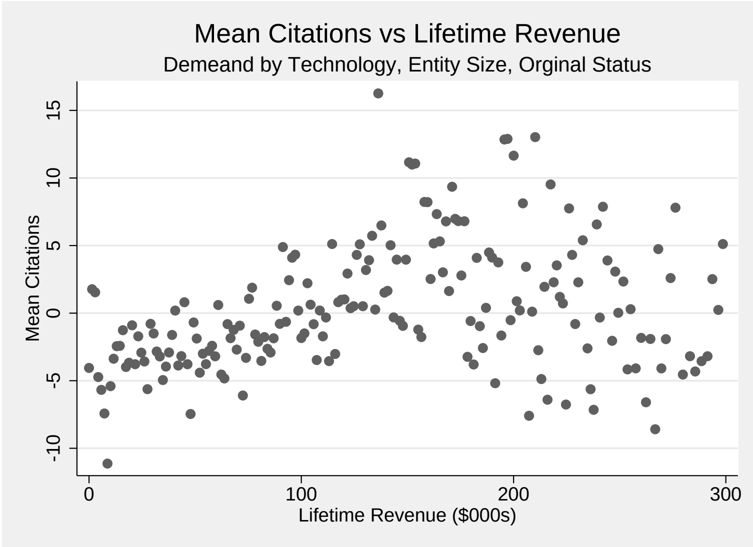

Figure 6: FORWARD CITATIONS VS. REVENUE (DEMEANED)
Notes: Data is normalized so that the mean annual revenue is $10,000.

value terms vary somewhat by which covariates are included, and individual inventor status seems to have the greatest impact. All specifications indicate a robust increasing and then decreasing relationship between forward citations and lifetime patent value. The final two columns of Table IV repeats this exercise but include the technological focus, breadth, a designation for filings in the U.S., patent family size, backward citations, claims, and dependent claims. Interestingly, number of claims and dependent claims are statistical insignificant, which echoes recent findings by Moser, Ohmstedt, and Rhode (2012). An Ear for Your Quotes: Patent Citations and the Size of Patented Inventions - Evidence from Hybrid CornThe main inverted-U relationship does not change even when many controls are included.

To this point, the reported data has largely bolstered the central empirical finding of the paper, of an inverted-U relationship between citations and value. But this sort of finding might be generated by a number of models of the innovation process. One of the important features of the model we propose is the decision of an innovator to engage in defensive patenting. This decision will vary by observable characteristics of the innovator and we may test the model's predictions empirically.

For example, since there is a fixed cost to defensive patenting, and firms tend to have greater resources than individuals on average, we would expect individuals to engage in less defensive patenting than companies. Since we have information on the original assignee of the patent in our data set, we can test this hypothesis. Figure 7 shows the relationship between the citations and patents, where the size and darkness of the data points is split into quartiles according to the corporate share of assignees. We find that the highest value patents, with

fewer citations also tend to be composed of a larger proportion of corporate assignees than the less-cited and less valuable patents.

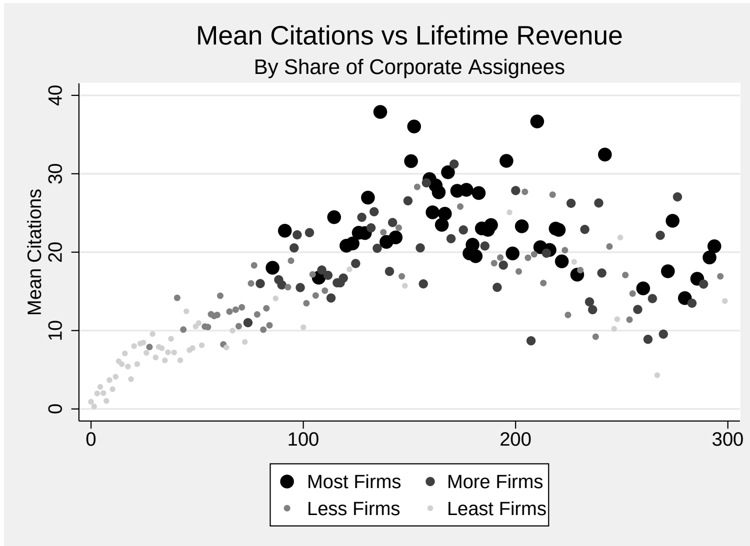

Figure 7: FORWARD CITATIONS VS. REVENUE BY SHARE OF CORPORATE ASSIGNEES

Notes: Data is normalized so that the mean annual revenue is $10,000.

Another patent characteristic that may influence likelihood of defensive patenting is the overall of level of innovation in the field. According to the model, defensive patenting will be more attractive if profits in a field are greater and this should be true for areas undergoing rapid development. One measure for the speed of development of a patent's field is the share of its backward citations within the prior several years. We would expect that patents with more recent backward citations are in fields of rapid growth, generating greater profits and thus incentives for defensive patenting.

Figure 8 reports the results of this analysis and is analogous to Figure 7. Now data point size and darkness is determined by its quartile of the distribution of the share of backward citations in the previous three years. As predicted by the model, we find those patents with the greatest share of recent backward citations to be toward the right end of the figure, with high revenues, but not particularly high citations. This is consistent with greater use of defensive patenting in fields of rapid innovation.

Figures 9 and 10 report the forward citation-patent value relationship in two additional ways, that provide further support to the theory of innovative and defensive patenting. Continuation and divisional patents are frequently employed strategically by sophisticated patentees in order to extend the duration of patent prosecution. This seems likely to be employed less frequently for truly innovative patents, because the value should be less dependent on market

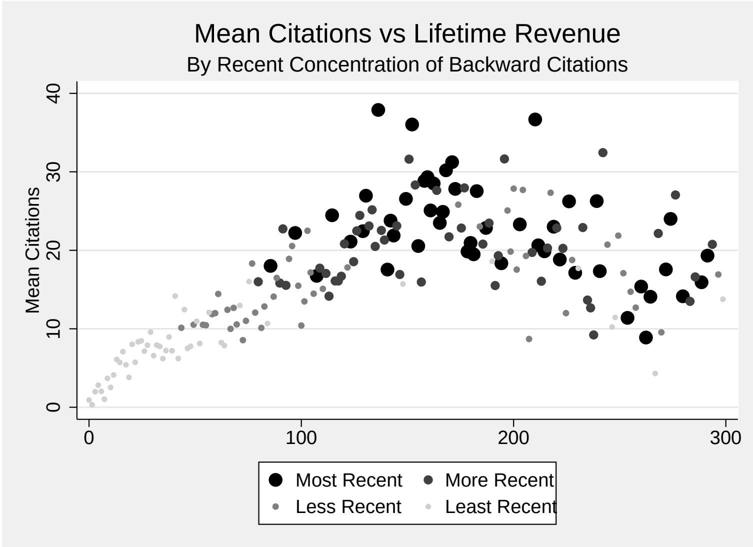

Figure 8: FORWARD CITATIONS VS. REVENUE BY RECENT CONCENTRATION OF BACKWARD CITATIONS

Notes: Data is normalized so that the mean annual revenue is $10,000.

conditions and thus extending patent prosecution have little value. In Figure 9 we observe that divisional and continuation patents are more prevalent in the high value/low citation region of the graph.

Dividing patents by their application date is useful to help investigate recent claims that defensive patenting has increased in recent years. We find evidence for this in Figure 10, which shows that the share of patents newer than the median is higher where revenues are higher and citations relatively lower. This does not provide an explanation for this trend, but does provide the first direct evidence of its existence.

Returning to the inverted-U relationship, Table V shows that the linear and quadratic values are generally consistent across each of the different technology categories, but for some the peaks are further out or the rate of decline is faster. The likely explanations for this observation are that the level of maturity of the technological innovation and the extent of cross-licensing and/or the decision to engage in defensive patenting vary across our 10 technology categories. For example, the inverted-U relationship remains strong in the semiconductor industry which is consistent with a prolonged period of defensive licensing among industry leaders such as Texas Instruments, IBM, AMD, and Samsung. In contrast, nanotechnology, which many consider a nascent technology, does not exhibit such an extensive bend. Figures 11 and 12 shows that the inverted-U relationship holds for software and computer architecture patents. This suggests that the relationship is not driven solely by technology distinctions such as software or hardware, but rather is likely due to the fact that both technology categories

are at more mature level in the innovation cycle.

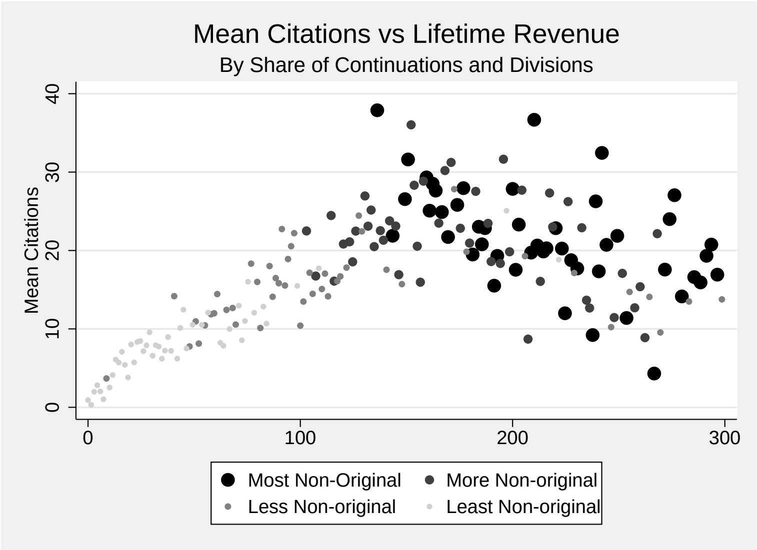

Figure 9: FORWARD CITATIONS VS. REVENUE BY SHARE OF CONTINUATIONS AND DIVISIONALS
Notes: Data is normalized so that the mean annual revenue is $10,000.

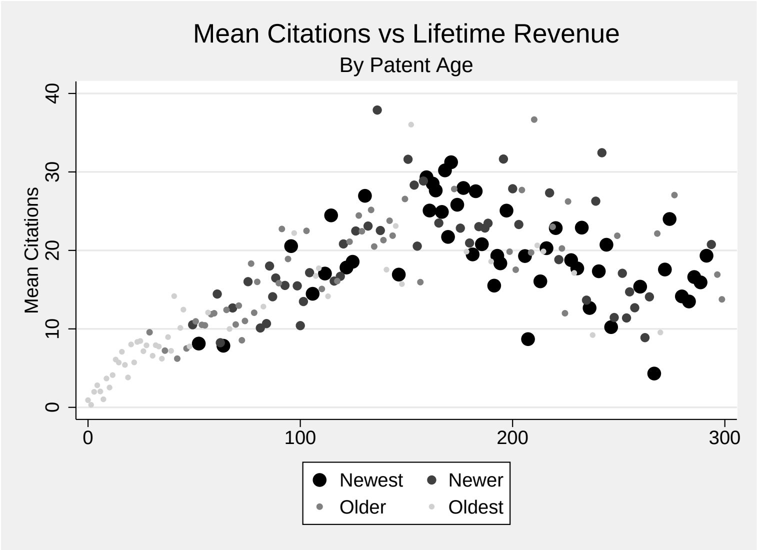

Figure 10: FORWARD CITATIONS VS. REVENUE BY PATENT AGE
Notes: Data is normalized so that the mean annual revenue is $10,000.

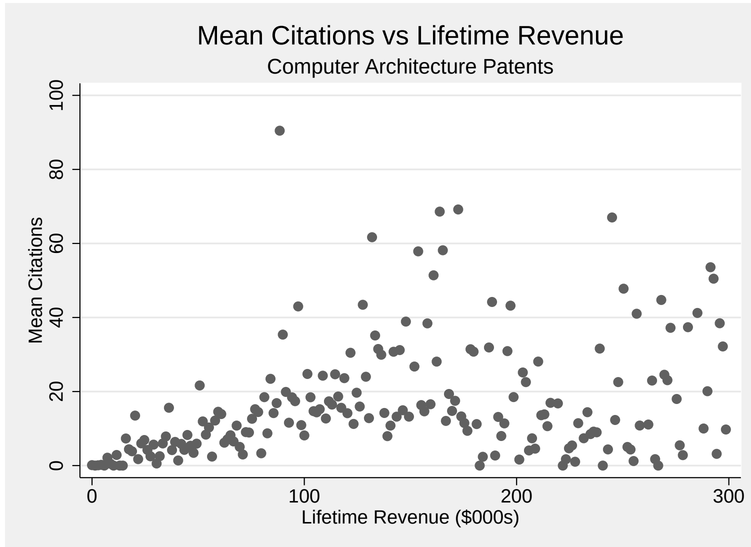

Figure 11: FORWARD CITATIONS VS. REVENUE, COMPUTER ARCHITECTURE PATENTS
Notes: Data is normalized so that the mean annual revenue is $10,000.

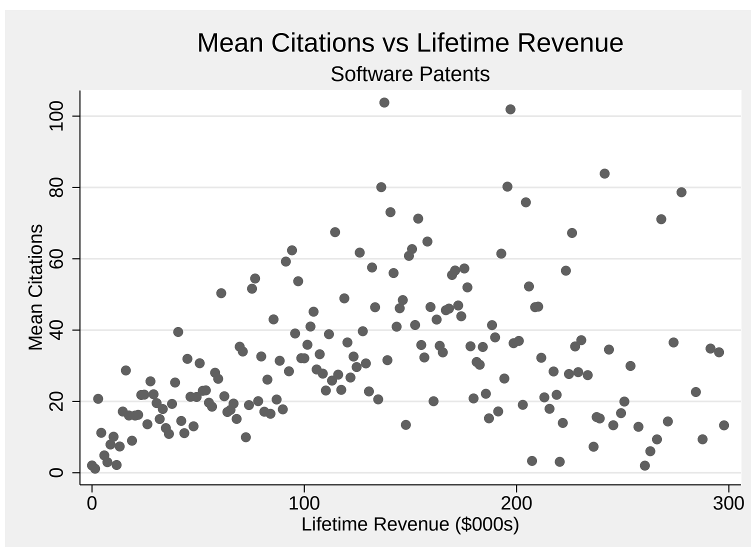

Figure 12: FORWARD CITATIONS VS. REVENUE, SOFTWARE PATENTS
Notes: Data is normalized so that the mean annual revenue is $10,000.

5 Conclusion

Using a new dataset with an unprecedented number of observations and patent-specific revenue we have found a surprising result for the relationship between forward citations and patent value. This finding should impact a large array of literature that has relied on citation-weighted patent counts to proxy for innovation. In forthcoming papers we hope to go beyond citations and propose new and better proxies for the value of innovation.

There are several concerns that may remain about the results presented here. The first is due to the fact that the data used is clearly a non-random sample of all patents; and thus, some of the results may be not be generalizable. This is certainly a concern and of course NPEs are in business to try to select the most valuable patents. It is unlikely that the main findings here are impacted much for at least two reasons. First, most NPEs do not use forward citations as an input into their decision-making process of which patents to purchase. Second, of the patents we observe in the portfolio, the vast majority was not a specific target by the NPEs for acquisition. Since NPEs target specific patents, of those additional patents that get included in the negotiated contract or that colloquially are "coming along for the ride," actually represent the majority of patents. Thus, taken as a whole, the patent portfolio can be viewed as closer to a random sample.

This leads to a second potential concern, that even if most of the patents examined were somewhat randomly selected, it is only from a pool of technology patents. This is clearly true, and the study is limited to the technology-related categories described in Table II. One might argue that this is not a shortcoming of the analysis, but a feature. Technology patents are an area of intense innovation and academic interest. Some subsets of technology patents, for example, software patents or those involved in smartphones are among the most controversial ones today. Thus it is particularly important to be able to understand more about the value of this field of innovation. It would, however, be a mistake to attempt to generalize the results found here to biotech patents, for example.

A final concern may be about the whether the model we put forth uniquely predicts the patterns we observe in the data. The basic inverted-U shape could no doubt be generated by a host of models of the innovative process. But the additional predictions and tests of the model, give us much greater confidence in its accuracy. We have seen that breaking up the data by individual inventor status, original versus continuation patents, age of patent and level of activity in a field all fall into patterns predicted by the model.

The real potential for this work is yet to come. Using the model introduced here creates the potential to rigorously analyze specific innovation-related policy proposals. If our understanding of the innovative process is correct, it will be able to guide decisions on questions such as broadening patent rights or increasing R&D subsidies.

There is also the potential for learning a great deal more about the innovation process,

but combining the data introduced here with further information about assignees, such as industry structure and concentration, corporate structure and history, and more. The goal of this line of work is to broaden and deepen our understanding of the innovation process, with an eye ultimately towards informing better policy decisions to foster it.

In this paper we have shown that the relationship between citations and patent value is not the simple one that has been assumed, but in fact takes on an "inverted-U" shape. We have found this using a new data set with patent-specific revenue for a large, diversified portfolio of technology patents. We model the innovation process in order to understand the origin of this complex relationship and find that two types of patents, innovative and defensive, not only explain the basic pattern observed, but predict other relationships in the data. We hope this paper will spur a resurgence of work attempting to better understand innovation informed by both theory and empirics.

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Tables

TABLE I — SUMMARY STATISTICS

	Mean	Std. Deviation	Median
Lifetime Revenue ($000s)	204.2	1904.7	52.19
Lifetime Forward Citations	13.4	38.4	0.0
Backward Citations	23.1	60.3	8.0
Concentrating of Backward Citations in Patents Applied for in Past 3 Years	20%	30%	0%
Concentrating of Backward Citations in Patents Applied for in Past 5 Years	28%	37%	0%
Original Indicator	84%	36%	1.00
Application Year	1999	4.7	2000
Individual Inventor Indicator	58%	49%	1.0

Notes: Data is normalized so that the mean annual revenue is $10,000. Forward citations are calculated as of the application date. Original patent applications establish their own filing date and do not have an effective filing date based upon another previously filed application. If an "original" application is then used to establish an effective filing date of a later filed application, it becomes known as a parent application and the later filings are either divisions or continuations.

TABLE II — SUMMARY OF TECHNOLOGIES REPRESENTED AND LIFETIME REVENUE

Technology	Mean Lifetime Revenue	Mean Lifetime Forward Citations
Peripheral Devices	99,801	8.1
Internet & Software	273,093	12.6
Semiconductor Devices	115,824	7.8
Wireless Communications	174,605	35.4
Network Communications	146,974	9.4
Optical Networking	56,425	16.5
Circuits	367,130	7.1
Computer Architecture	283,773	6.0
Electro-Mechanical	62,018	7.4
MEMS & Nano	58,860	11.1

Notes: Data is normalized so that the mean annual revenue is $10,000. To calculate lifetime revenue, for each technology category, we create a profile of expected mean revenue by patent age, where age equal to 0 is defined as the application year. Using all years of annual revenue data available, 2008-2012, we calculate the aggregate revenue through 2012 and the age of the patent at the end of 2012. We match our observed revenue profile for each patent to the expected profile. The ratio of the expected to the observed is used to estimate the lifetime revenue.

TABLE III — NON-LINEAR RELATIONSHIP BETWEEN LIFETIME CITATIONS AND LIFETIME REVENUE

	Excluding Top Percent of Revenue Distribution		
	10%	5%	1%
Lifetime Revenue	22.497	14.402	8.016
	(34.41)**	(25.45)**	(18.57)**
Lifetime Revenue Squared	-6.036	-2.193	-0.139
	(20.95)**	(11.25)**	(1.98)*
Constant	0.941	2.917	5.035
	(5.70)**	(15.94)**	(22.10)**
R^2	0.05	0.05	0.09
N	42,878	44,634	46,329

** Significant at the 1 % level; * Significant at the 5 % level

Notes: Data is normalized so that the mean annual revenue is $10,000. To calculate lifetime revenue, for each technology category, we create a profile of expected mean revenue by patent age, where age equal to 0 is defined as the application year. Using all years of annual revenue data available, 2008-2012, we calculate the aggregate revenue through 2012 and the age of the patent at the end of 2012. We match our observed revenue profile for each patent to the expected profile. The ratio of the expected to the observed is used to estimate the lifetime revenue.

TABLE IV — RELATIONSHIP AFTER CONDITIONING FOR OTHER PATENT CHARACTERISTICS

	1	2	3	4	5	6
Lifetime Revenue	7.569	9.272	8.669	8.444	5.18	5.544
	(12.17)**	(14.55)**	(13.73)**	(13.73)**	(8.15)**	(5.37)**
Lifetime Revenue Squared	-0.906	-1.254	-1.213	-1.13	-0.666	-0.648
	(4.41)**	(6.09)**	(5.89)**	(5.63)**	(3.29)**	(2.30)*
Individual Inventor	-18.512	-18.364	-17.141	-17.209	-1.9	3.966
	(47.76)**	(47.66)**	(42.25)**	(43.10)**	(2.30)*	(3.19)**
Patent Application Before 2000		5.347	5.968	6.337	8.025	18.279
		(16.11)**	(18.06)**	(19.07)**	(21.70)**	(25.41)**
Indicator Original Patent			-7.583	-5.384	3.414	3.896
			(11.12)**	(8.17)**	(4.44)**	(4.63)**
Tech Category (Computer Architecture)				3.632	2.934	5.591
				(6.43)**	(4.95)**	(5.42)**
Tech Category (Electro-Mechanical)				4.03	3.044	6.39
				(6.28)**	(4.31)**	(4.63)**
Tech Category (Internet & Software)				19.87	16.733	26.894
				(22.78)**	(19.33)**	(19.89)**
Tech Category (MEMS & Nano)				3.798	1.723	3.107
				(2.89)**	-1.09	-1.32
Tech Category (Networking & Communications)				9.808	9.598	19.199
				(13.37)**	(12.17)**	(13.17)**
Tech Category (Optical Networking)				2.1	0.226	0.846
				(4.45)**	-0.45	-0.89
Tech Category (Peripheral Devices)				2.508	2.534	4.076
				(6.07)**	(5.79)**	(4.99)**
Tech Category (Semiconductors)				3.387	3.88	6.141
				(7.86)**	(8.54)**	(6.69)**
Tech Category (Wireless Communications)				7.22	5.884	12.188
				(13.79)**	(10.95)**	(12.07)**
Technological Focus					0.6	0.574
					(3.82)**	(3.61)**
Technological Breadth					4.125	2.408
					(10.08)**	(5.45)**
Indicator US Patent					16.517	10.841
					(26.73)**	(18.41)**
Patent Family Size					0.051	0.175
					(3.04)**	(4.86)**
Backward Citations					0.05	0.024
					(4.12)**	-1.95
Claims						0.196
						-0.97
Dependent Claims						0.172
						-0.79
Constant	18.292	14.533	20.389	12.207	-13.815	-23.427
	(42.93)**	(31.19)**	(28.98)**	(15.28)**	(10.11)**	(13.04)**
R2	0.12	0.12	0.13	0.16	0.21	0.14
N	40,257	40,257	40,257	40,257	36,893	18,715

** Significant at the 1 % level; * Significant at the 5 % level

Notes: Data is normalized so that the mean annual revenue is $10,000.

TABLE V — RELATIONSHIP BETWEEN LIFETIME CITATIONS AND LIFETIME REVENUE BY TECHNOLOGY

	Circuits	Computer Architecture	Electro-Mechanical	Internet & Software	MEMS & Nano
Lifetime Revenue	6.233	14.497	10.917	23.542	17.051
	(6.89)**	(11.28)**	(6.60)**	(10.95)**	(4.75)**
Lifetime Revenue Squared	-0.777	-2.212	-2.341	-3.184	-4.325
	(3.18)**	(6.27)**	(3.93)**	(4.39)**	(3.80)**
Constant	2.706	1.601	3.666	9.859	2.566
	(6.77)**	(3.30)**	(5.78)**	(11.71)**	(2.40)*
R^2	0.05	0.09	0.04	0.05	0.06

	Networking Communication	Optical Networking	Peripheral Devices	Semiconductors	Wireless Communications
Lifetime Revenue	19.107	13.496	9.847	9.329	18.007
	(8.64)**	(11.43)**	(14.64)**	(9.60)**	(12.04)**
Lifetime Revenue Squared	-2.328	-2.114	-2.355	-1.020	-3.292
	(2.90)**	(4.57)**	(11.09)**	(3.01)**	(5.91)**
Constant	2.859	1.533	2.462	1.678	1.225
	(4.06)**	(4.26)**	(10.19)**	(5.37)**	(2.84)**
R^2	0.08	0.07	0.02	0.06	0.07

** Significant at the 1 % level; * Significant at the 5 % level

Notes: Data is normalized so that the mean annual revenue is $10,000.